Section 7.3

Definition: A **probability distribution** is an assignment of a number $P(s_i)$, the **probability of s_i**, to each outcome of a finite sample space $S = \{s_1, s_2, \ldots, s_n\}$. The probabilities must satisfy

1. $0 \leq P(s_i) \leq 1$
2. $P(s_1) + P(s_2) + \cdots + P(s_n) = 1$.

We find the **probability of an event E,** written $P(E)$ by adding up the probabilities of the outcomes in E. If $P(E) = 0$, we call E and **impossible event**. The empty set event $\emptyset$ is always impossible, since *something* must happen.

Definition: A **probability model** for a particular experiment is a probability distribution that predicts the relative frequency of each outcome if the experiment is performed a large number of times. Just as we think of relative frequency as *estimated probability*, we can think of modeled probability as *theoretical probability*.

Probability Model for Equally Likely Outcomes: In an experiment in which all outcomes are equally likely, we model the experiment by taking the probability of an experiment to be

$$P(E) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{n(E)}{n(S)}.$$

Addition Principle: If A and B are any two events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. If $A \cap B = \emptyset$, we say that A and B are **mutually exclusive**, and we have $P(A \cup B) = P(A) + P(B)$.

More Principles of Probability Distributions: The following rules hold for any sample space S and any event A:

$P(S) = 1$ (The probability of *something* happening is 1)
$P(\emptyset) = 0$ (The probability of *nothing* happening is 0)
$P(A') = 1 - P(A)$ (The probability of A *not* happening is 1 minus the probability of A)

Problem 1. Complete the following probability distribution table and then calculate the stated probabilities.

Outcome	a	b	c	d	e
Probability	.1		.65	.1	.05

a. $P(\{\text{a, c, e}\})$

b. $P(E \cup F)$, where $E = \{\text{a, c, e}\}$ and $F = \{\text{b, c, e}\}$

c. $P(E')$, where E is as in part (b)

d. $P(E \cap F)$, where E and F are as in part (b).

Problem 2. Calculate the (modeled) probability $P(E)$ using the given information, assuming that all outcomes are equally likely.

a. $n(S) = 8, \;\; n(E) = 4$

b. $S = \{1, 3, 5, 7, 9\}, \quad E = \{3, 7\}$

Problem 3. An experiment is given together with an event. Find the (modeled) probability of each event, assuming that the coins and dice are distinguishable and fair.

a. Two coins are tossed; the result is one or more heads.

b. Three coins are tossed; the result is more tails than heads.

c. Two dice are rolled; the numbers add to 9.

d. Two dice are rolled; one of the numbers is even, the other is odd.

Problem 4. If two indistinguishable dice are rolled, what is the probability of the event $\{(4, 4), (2, 3)\}$? What is the corresponding event for a pair of distinguishable dice?

Problem 5. A die is weighted in such a way that each of 1 and 2 is three times as likely to come up as each of the other numbers. Find the probability distribution. What is the probability of rolling an even number?

Problem 6. Use the given information to find the indicated probability.

a. $P(A) = .3, P(B) = .4, P(A \cap B) = .02$. Find $P(A \cup B)$.

b. $A \cap B = \emptyset, P(B) = .8, P(A \cup B) = .8$. Find $P(A)$.

c. $P(A \cup B) = 1.0, P(A) = .6, P(A \cap B) = .1$. Find $P(B)$.

d. $P(A) = .22$. Find $P(A')$.

Problem 7. Determine whether the information shown is consistent with a probability distribution. If not, say why.

a. $P(A) = .2; P(B) = .4; P(A \cup B) = .2$

b. $P(A) = .2; P(B) = .4; P(A \cap B) = .3$

Problem 8. The following table shows the profile, by the math section of the SAT Reasoning Test, of admitted students at UCLA for the fall 2011 semester.

SAT Reasoning Test—Math Section

	700–800	600–699	500–599	400–499	200–399	Total
Admitted	6,611	3,981	1,388	385	8	12,373
Not admitted	6,622	13,876	9,890	4,974	1,363	36,725
Total applicants	13,233	17,857	11,278	5,359	1,371	49,098

Determine the probabilities of the following events (round answers to the nearest .01).

a. An applicant had a Math SAT below 400.

b. An applicant had a Math SAT of 700 or above and was admitted.

c. An applicant did not have a Math SAT below 400.

d. An applicant had a Math SAT of 700 or above or was admitted.

Problem 9. According to a *New York Times*/CBS poll of March 2005, 49% agreed that Social Security taxes should be raised if necessary to keep the system afloat, and 43% agreed that it would be a good idea to invest part of their Social Security taxes on their own. What is the largest percentage of people who could have agreed with at least one of these statements? What is the smallest percentage of people who could have agreed with at least one of these statements?

Problem 10. Lance the Wizard has been informed that tomorrow there will be a 50% chance of encountering the evil Myrmidons and a 20% chance of meeting up with the dreadful Balrog. Moreover, Hugo the Elf has predicted that there is a 10% chance of encountering both tomorrow. What is the probability that Lance will be lucky tomorrow and encounter neither the Myrmidons nor the Balrog?

Homework: Read section 7.3, do #1, 3, 7, 13, 19, 21, 31, 71, 75